# CU06997 Fluid Dynamics / CU03287 Waterstroming: formulas

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# Principal symbols / units

Wetted Area [m2] Natte doorsnede  
 Cross-sectional area of flow

b = width [m] Breedte

C = Chézy coefficient [m1/2/s] Coefficient van Chezy

velocity coefficient [-] Snelheidscoëfficiënt  
 contraction coefficient [-] Contractiecoëfficiënt

d, D = diameter [m] Diameter

Dm = Hydraulic mean depth [m] Gemiddelde hydraulische diepte

E = Energy [J] =[Nm] Energie

Es = specific energy [m] Specifieke energie

F = Force [N] Kracht

Fr = Froude Number [-] Getal van Froude

g = gravitational acceleration [m/s2] Valversnelling

H = head [m] Energiehoogte

hf , ∆H = frictional head loss [m] Energieverlies tgv wrijving

hL = local head loss [m] Lokaal energieverlies

kL = local loss coefficient [-] Lokaal energieverlies coëfficiënt

kS = surface roughness [m] Wandruwheid

L = length [m] Lengte

m = mass [kg] Massa

n = Manning’s roughness coefficient [s/m1/3] Coëfficiënt van manning

p\* = piezometric pressure [N/m2]= [Pa] Piezometrische druk

p = pressure [N/m2] Druk

P = wetted perimeter [m] Natte omtrek

Ps = crest height [m] Stuwhoogte

Q = discharge, flow rate [m3/s] Debiet, afvoer

q = discharge per unit channel width [m3/ms] Debiet per m breedte

R, r = radius [m] Straal

R = Hydraulic radius [m] Hydraulische straal

Re = Reynolds Number [-] Getal van Reynolds

Sc = slope of channel bed to give critical flow [-] Bodemverhang voor grenssnelheid

Sf ,I = slope of hydraulic gradient [-] Energieverhang

S0 = slope of channel bed [-] Bodemverhang

Ss = slope of water surface [-] Drukverhang, verhang water

u,v = velocity [m/s] Stroomsnelheid

V = mean velocity [m/s] Gemiddelde stroomsnelheid

V = volume [m3] Volume

ū = average velocity [m/s] Gemiddelde stroomsnelheid

y = water depth [m] Waterdiepte

yc = critical depth [m] Kritische waterdiepte

yn = normal depth [m] Normale waterdiepte

z = height above datum, potential head [m] Afstandshoogte

δ = boundary layer thickness [m] Dikte grenslaag

λ = friction factor [m] Wrijvingsfaktor

µ = absolute viscosity [kg/ms]=[N s/m2] Absolute viscositeit

ν = kinematic viscosity [m2/s] Kinematische viscositeit

ρ = density of liquid [kg/m3] Soortelijk gewicht

τ0 = shear stress at solid boundary [N/m2] Schuifspanning  
ξ = (ksie) Loss coefficient [1] Verliescoëfficiënt  
µ = contraction coefficient [1] Contractiecoëfficiënt

# Fluid statics

General pressure intensity

Pressure [Pa=N/m2]  
 Force [N]  
 Area on which the force acts [m2]

Newton Force

Force [N]  
 Weight [Kg]  
 earths gravity [m/s2]

Fluid Pressure at a point

Pressure Head

Pressure [Pa=N/m2]  
 fluid density [Kg/m3]  
 earths gravity [m/s2]  
 distance surface to point [m]

[Kg/m3]  
 [Kg/m3]

Potential Head

height above datum [m]

Piezometric Head

height above datum [m]  
 distance surface to point [m]

Velocity Head

Fluid Velocity [m/s]  
 earths gravity [m/s2]

# Visualisation flow, streamlines, streaklines, streamtube

Flow rate / Discharge

Flow rate [m3/s]  
 Fluid Velocity [m/s]  
 Wetted Area [m2]

Wetted Area of a filled pipe   
 Wetted Area [m2]  
 Diameter pipe [m2]

Continuity equation   
(Principle of conservation of mass)

# Total Head or Bernoulli’s Equation

Total Head / Energy [m]

Pressure Head [m]  
 Potential Head [m]  
 Velocity Head [m]

Bernoulli’s Equation without head loss

Bernoulli’s Equation with head loss

Pressure Head [m]  
 Potential Head [m]  
 Velocity Head [m]  
 Head Loss [m]  
 earths gravity [m/s2]

Momentum equation

Force [N]  
 fluid density [Kg/m3]  
 Flow rate [m3/s]  
 Mean velocity before [m/s]  
 Mean velocity after [m/s]

Pitot

Fluid Velocity [m/s]  
 earths gravity [m/s2]  
 Difference in pressure [m]

Discharge small orifice

Flow rate [m3/s]  
 Wetted Area [m2]  
 velocity coefficient (0,97-0,99) [-]  
 contraction coefficient (0,61-0,66) [-]  
 earths gravity [m/s2]  
 Difference in pressure [m]  
Discharge large orifice

Flow rate [m3/s]  
 Width orifice [m2]  
 earths gravity [m/s2]  
 Difference in pressure from top [m]  
 Difference in pressure from bottom [m]

# Turbulent and Laminar flow, Reynolds Number

Kinematic viscosity

Absolute viscosity [kg/ms]   
 Kinematic viscosity [m2/s] water, 20°C=   
 Density of liquid [kg/m3]

Reynolds Number

Kinematic viscosity [m2/s] water, 20°C=   
 Density of liquid [kg/m3]   
 Mean velocity [m/s]  
R = Hydraulic Radius = D/4 [m]  
 Reynolds Number [1]

Turbulent flow  
Laminar flow

Hydraulic Radius

Hydraulic Radius [m]  
 Wetted Area [m2]  
 Wetter Perimeter [m]

Hydraulic radius of a filled pipe

Hydraulic radius of a 50% filled pipe

Hydraulic Diameter

Hydraulic Radius [m]  
 Hydraulic Diameter [m]

# Laminar flow in pipes and closed conduits

Frictional head loss (laminar flow)

= frictional head loss [m] Absolute viscosity [kg/ms]   
 Length between the Head Loss [m]  
 mean velocity [m/s]  
D = Hydraulic Diameter [m]  
 Density of liquid [kg/m3]   
 earths gravity [m/s2]

Wall shear stress (laminar flow)

τ0 = shear stress at solid boundary [N/m2]  
 Absolute viscosity [kg/ms]   
 mean velocity [m/s] Hydraulic Radius [m]

# Turbulent flow in pipes and closed conduits

Head loss / Energy loss

Head Loss [m]   
 Velocity Head [m]  
 Loss coefficient [1]  
 earths gravity [m/s2]

## Frictional head losses

Darcy-Weisbach

Head Loss due to friction [m]   
 Friction coefficient [1]  
 Velocity Head [m]  
D = Hydraulic Diameter 4R [m]  
 Length between the Head Loss [m]  
 earths gravity [m/s2]

Colebrook-White transition formula

Friction coefficient [1]  
D = Hydraulic Diameter [m]  
*k*S = surface roughness [m]

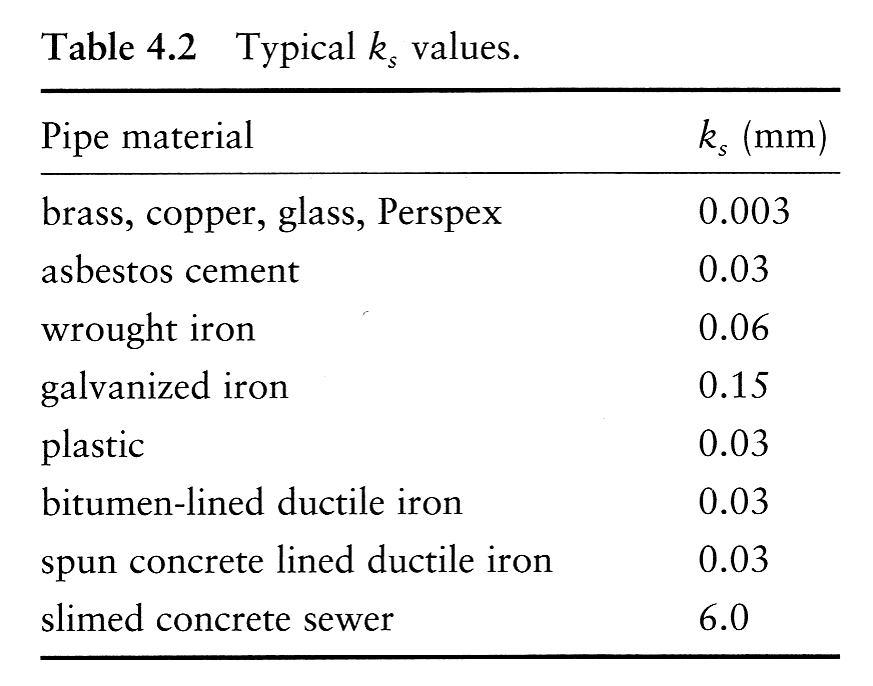
Colebrook-White and Darcy Weisbach

with

mean velocity [m/s]

D = Hydraulic Diameter [m]  
*k*S = surface roughness [m] Kinematic viscosity [kg/ms] water, 20°C=   
Sf = slope of hydraulic gradient [-]

*hf* = frictional head loss [m] Length between the Head Loss [m]  
 earths gravity [m/s2]



## Local head losses

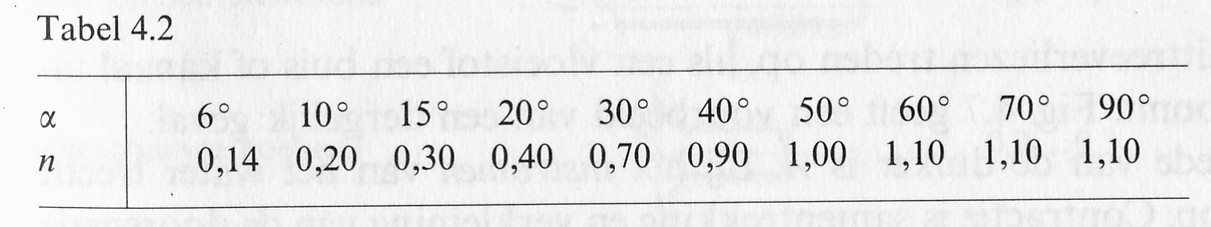
Sudden Pipe Enlargement

Head Loss due to sudden pipe enlargement [m]   
 Loss coefficient due to sudden pipe enlargement [1]  
 Wetted Area [m2]  
 Mean Fluid Velocity [m/s]  
 earths gravity [m/s2]  
1= Before enlargement  
2= After enlargement

Sudden Pipe Contraction

Head Loss due to sudden pipe contraction [m]   
 Mean Fluid Velocity after sudden pipe contraction [m/s]   
 earths gravity [m/s2]

Tapered Pipe Enlargement

Head Loss due to tapered pipe enlargement [m]   
 Loss coefficient due to tapered pipe enlargement [1]  
 Wetted Area [m2]  
1= Before enlargement  
2= After enlargement  
 factor which depends on the widening angel α  


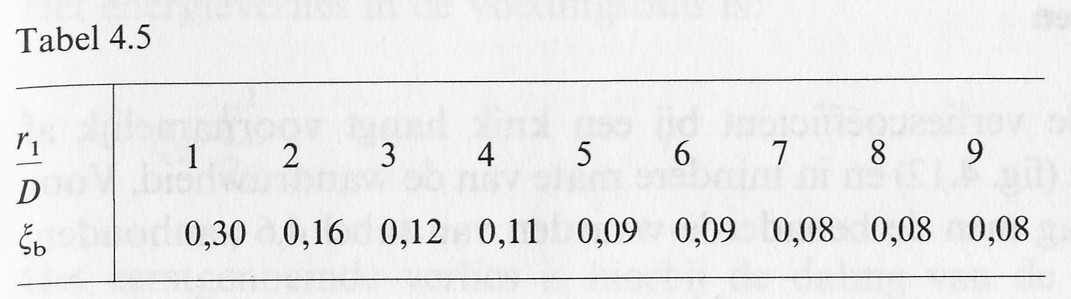
Submerged Pipe Outlet

Head Loss due to submerged pipe outlet [m]   
 Mean Fluid Velocity before pipe outlet [m/s]   
 Loss coefficient due to submerged pipe outlet [1]  
 earths gravity [m/s2]

Pipe Bends

Head Loss due to pipe bend [m]   
 Mean Fluid Velocity [m/s]   
 Loss coefficient due to pipe bend [1]  
 earths gravity [m/s2]

|  |  |
| --- | --- |
| D:\CourseVloeistofmechanica\Lesblok 6 en 7\fig4.10.jpg | Tabel 4.5 only applies for α = 90o and a smooth pipe.  With α = 90o and a rough pipe, increase ξ with 100%  With α = 45o use 75% ξ900  With α = 22,5o use 50% ξ900  Smooth and rough pipes are explained further on. |



## Partially full pipes

Wetted Area partially filled pipe [m2]  
 Hydraulic radius partially filled pipe [m]  
h = water level partially filled pipe [m]  
D = Diameter pipe [m]

## Culverts

Culvert submerged 1   
   
   
   
 

Total Head Loss Culvert [m]   
 Sum of Loss coefficients [1]  
 Mean Fluid Velocity Culvert [m/s]   
 Loss coefficient due to contraction [1]  
 Loss coefficient due to friction [1]  
 Loss coefficient due to outlet [1]  
 Contraction coefficient [1]  
 earths gravity [m/s2]  
 Friction coefficient [1]  
 Hydraulic Radius [m]  
 Length between the Head Loss [m]

Culvert submerged 2   
   
 Flow rate Culvert [m3/s]  
 Discharge coefficient [m]  
 Wetted Area Culvert [m2]  
 Total Head Loss Culvert [m]   
 Sum of Loss coefficients [1]  
 earths gravity [m/s2]

Culvert partly submerged

**Free flow broad crested weir**  
(Volkomen lange overlaat)   
 

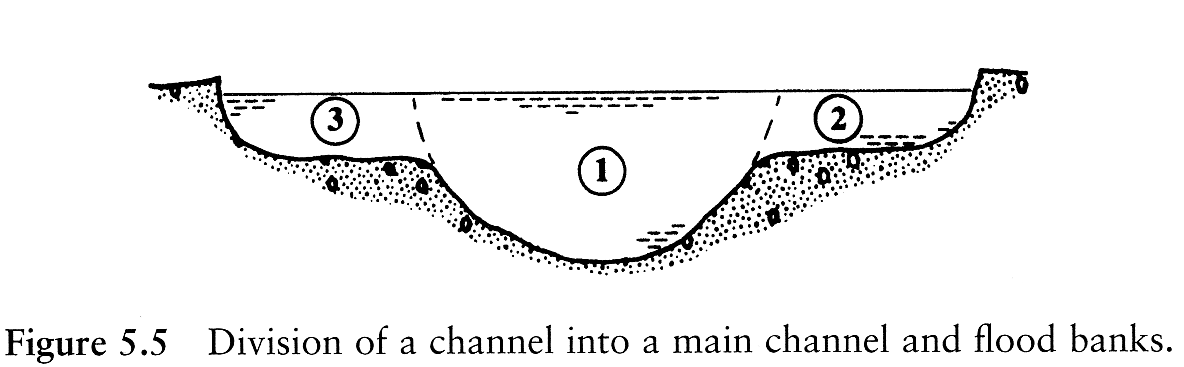
Discharge Culvert [m3/s]  
 Width weir [m]  
cv=discharge coefficient free flow broad crested weir [m1/2/s]

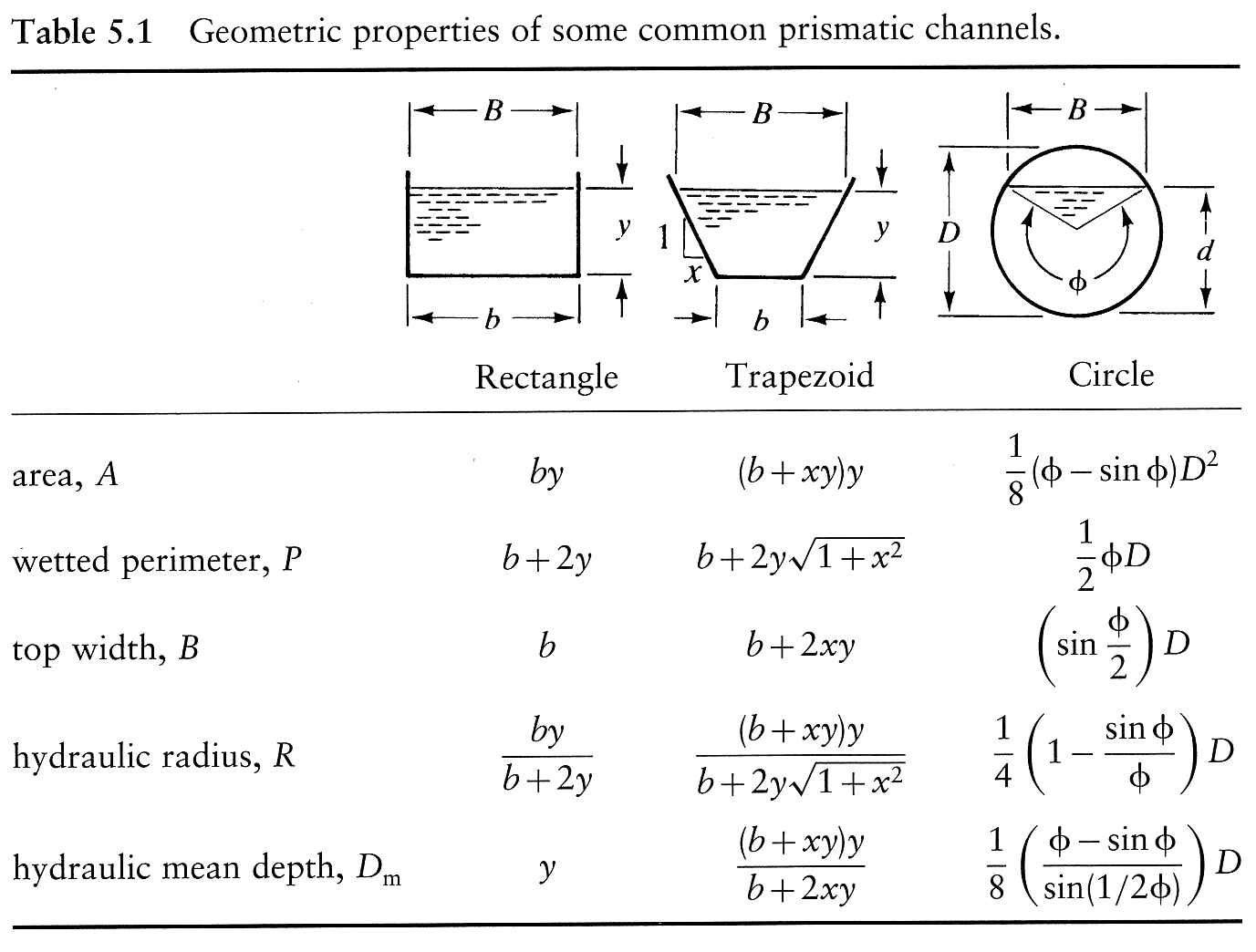
Head Loss upstream [m]   
 Water level downstream [m]

**Submerged flow broad crested weir**  
(Onvolkomen lange overlaat)   
   
   
 Discharge Culvert [m3/s]  
 Width weir [m]  
col=discharge coefficient submerged flow broad crested weir [1]  
 Head Loss upstream [m]   
 Water level downstream [m]

Total head (H) and water level (h) measured from crest weir (bed culvert)

# Open channel flow





## Frictional head losses, turbulent flow

Mean boundary shear stress

τ0 = shear stress at solid boundary [N/m2]  
 Hydraulic Radius [m]  
 Slope of channel bed [1]

Chezy

Mean Fluid Velocity [m/s]  
 Hydraulic Radius [m]  
 Slope energy / total head [1]  
 Chezy coefficient [m1/2/s]

Manning

Mean Fluid Velocity [m/s]  
 Hydraulic Radius [m]  
 Slope Total Head [1]  
 Wetted Area [m2]  
 Wetter Perimeter [m]  
 Mannings roughness coefficient [s/m1/3]

Specific energy

Mean Fluid Velocity [m/s]  
 Pressure Head / water depth [m]

Equilibrium / normal depth [m]

*y*n = normal depth, equilibrium depth [m]

q = discharge [m3/s]

b = width [m]

bed slope [1]  
 Hydraulic gradient caused by friction [1]  
 Chezy coefficient [m1/2/s]

Backwater, direct step method

Δx= horizontal distance from point [m]

Δy= waterdepth [m]

Fr = Froude number [-]

bed slope [1]  
 Hydraulic gradient caused by friction [1]

## Subcritical and Supercritical flow

Critical depth [m]

Critical velocity [m/s]

Froude Number

*y*c = critical depth [m]

Q = discharge [m3/s]

B = width [m]

*V*c = critical velocity [m/s]

*V* = actual velocity [m/s]

Fr = Froude number [-]

**Subcritical flow** Fr < 1 V < Vc

**Supercritical flow** Fr > 1 V > Vc

Energy loss hydraulic jump

ΔH = Energy loss hydraulic jump [m]  
*y*1= depth supercritical flow [m]

*Y*2= depth subcritical flow [m]

Critical bed slope

*S*c = critical bed slope [-]

*y*c = critical depth [m]

Mannings roughness coefficient [s/m1/3]

## Hydraulic structures

Thin plate (sharp crested weirs)

Rehbock formula Q

Q = discharge [m3/s]

b = width [m] *h1* = pressure above crest [m]

*Ps* = crest height [m]

Vee weirs

Q = discharge [m3/s]

discharge coefficient [-] θ=90°, Cd=0.59

*h1* = pressure above crest [m]

θ = angle vee [°]

Rectangular broad crested weir

Ackers

Q = discharge [m3/s]

b = width [m] *h1* = pressure above crest [m]

*Ps* = crest height [m]

L = length weir [m]

Cf = friction coefficient [-]

Discharge broad-crested weir

Q = discharge [m3/s]

b = width [m] velocity coefficient ) [-]  
 discharge coefficient [-]

h = water pressure above crest [m]

Venturi flume

Q = discharge [m3/s]

b = width [m]   
 velocity coefficient ) [-]  
 discharge coefficient [-]

*y1* = pressure above crest [m]

# Sewers

Filled pipes

Head Loss, energy loss [m]

Q = discharge pipe [m3/s]

L = length of the pipe [m]

Chezy coefficient [m1/2/s]  
 Hydraulic Radius [m]  
 Wetted Area, flow surface [m2]  
Chezy coefficient

Chezy coefficient [m1/2/s]  
 Hydraulic Radius [m]  
*k*S = surface roughness [m]

Energy Gradient [-]

Sf ,i = slope of hydraulic gradient [-]

L = length [m]

Head Loss, energy loss [m]

Overflow / weir sewer

Q = discharge overflow [m3/s]

m = runoff coefficient (1,5 – 1,8) [m1/2/s]

B = Width crest overflow [m]

H = Head at overflow [m]  
 measured from top crest!!